

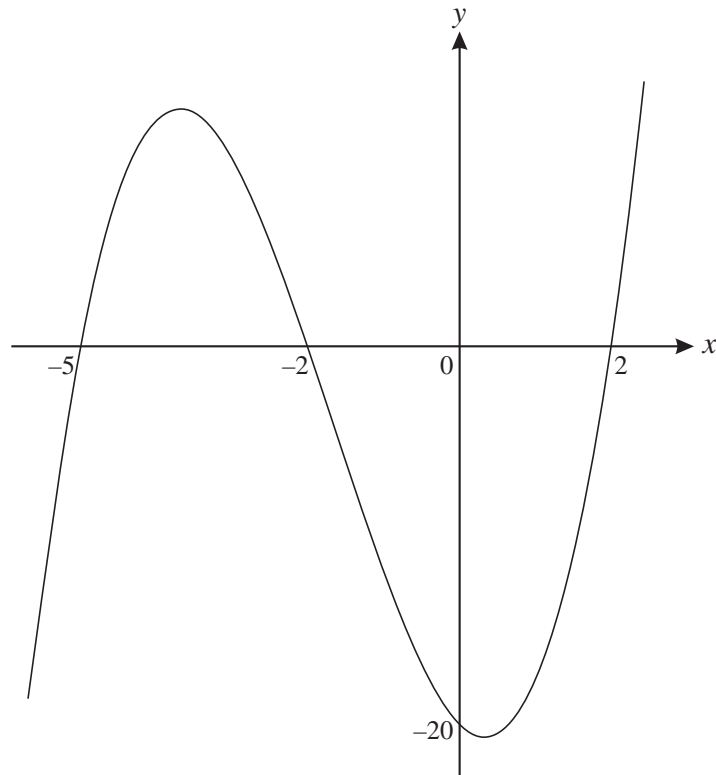
- 1 (i) Use calculus to find, correct to 1 decimal place, the coordinates of the turning points of the curve  $y = x^3 - 5x$ . [You need not determine the nature of the turning points.] [4]
- (ii) Find the coordinates of the points where the curve  $y = x^3 - 5x$  meets the axes and sketch the curve. [4]
- (iii) Find the equation of the tangent to the curve  $y = x^3 - 5x$  at the point  $(1, -4)$ . Show that, where this tangent meets the curve again, the  $x$ -coordinate satisfies the equation

$$x^3 - 3x + 2 = 0.$$

Hence find the  $x$ -coordinate of the point where this tangent meets the curve again. [6]

2 The equation of a cubic curve is  $y = 2x^3 - 9x^2 + 12x - 2$ .

- (i) Find  $\frac{dy}{dx}$  and show that the tangent to the curve when  $x = 3$  passes through the point  $(-1, -41)$ . [5]
- (ii) Use calculus to find the coordinates of the turning points of the curve. You need not distinguish between the maximum and minimum. [4]
- (iii) Sketch the curve, given that the only real root of  $2x^3 - 9x^2 + 12x - 2 = 0$  is  $x = 0.2$  correct to 1 decimal place. [3]



**Fig. 11**

Fig. 11 shows a sketch of the cubic curve  $y = f(x)$ . The values of  $x$  where it crosses the  $x$ -axis are  $-5$ ,  $-2$  and  $2$ , and it crosses the  $y$ -axis at  $(0, -20)$ .

- (i) Express  $f(x)$  in factorised form. [2]
- (ii) Show that the equation of the curve may be written as  $y = x^3 + 5x^2 - 4x - 20$ . [2]
- (iii) Use calculus to show that, correct to 1 decimal place, the  $x$ -coordinate of the minimum point on the curve is  $0.4$ .

Find also the coordinates of the maximum point on the curve, giving your answers correct to 1 decimal place. [6]

- (iv) State, correct to 1 decimal place, the coordinates of the maximum point on the curve  $y = f(2x)$ . [2]

- 4 (i) Differentiate  $x^3 - 3x^2 - 9x$ . Hence find the  $x$ -coordinates of the stationary points on the curve  $y = x^3 - 3x^2 - 9x$ , showing which is the maximum and which the minimum. [6]
- (ii) Find, in exact form, the coordinates of the points at which the curve crosses the  $x$ -axis. [3]
- (iii) Sketch the curve. [2]

5 The equation of a curve is  $y = 7 + 6x - x^2$ .

- (i) Use calculus to find the coordinates of the turning point on this curve.

Find also the coordinates of the points of intersection of this curve with the axes, and sketch the curve. [8]

- (ii) Find  $\int_1^5 (7 + 6x - x^2) dx$ , showing your working. [3]

- (iii) The curve and the line  $y = 12$  intersect at  $(1, 12)$  and  $(5, 12)$ . Using your answer to part (ii), find the area of the finite region between the curve and the line  $y = 12$ . [1]